Elements 15: 405-410 Supplementary Material:

Kimberlite Volcanology: Transport, Ascent and Eruption

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ENTRAINMENT AND MODIFICATION OF MANTLE CARGO

Decompression during kimberlite ascent induces internal stresses in xenoliths requiring volume expansion. High ascent rates imply rapid decompression rates that will exceed normal rates of ductile relaxation in xenoliths. The internal elastic stresses will accumulate promoting tensile failure to produce smaller xenoliths and xenocrysts. Residual stresses ($\Delta\sigma_R$) depend on ascent velocity (U), the bulk modulus (K_T) and viscosity (μ) of the xenolith, and the pressure drop over the ascent distance [$z - z_0$]:

$$\Delta\sigma_R = \rho\, g\, [z_0 - z] - K_T \frac{\rho g}{2\, \mu\, U} [z_0 - z]^2 \ . \ \text{(S1)}$$

The velocity of the ascending magma dictates the time available for viscous relaxation. Equation S1 models the stress state mantle material (xenoliths and xenocrysts) as a function of transport distance (i.e. decompression) in an ascending magma as a function of ascent velocities. These stresses can then be compared to the tensile strengths of coarse and fine-grained peridotite or olivine grains.

Faster ascent rates dictate less viscous relaxation causing failure at smaller transport distances. Slower ascent allows longer times for viscous relaxation and therefore longer transport distances before olivine, for example, fractures. Any ascent velocity ≥1 m s^{-1} is sufficiently rapid to suppress viscous relaxation and the residual stresses are equal to the elastic limiting stresses. At these velocities the internal stresses in olivine rise to exceed the tensile strength of olivine after ~15 km of ascent (~4 h at 1 m s^{-1}); at these velocities the magma would transit 200 km in ~2 days. At lower velocities (e.g., 0.1 m s^{-1}), larger transport distances (22 km) are required to generate internal residual stresses exceeding the tensile strength of olivine.

Polymineralic rocks (e.g. peridotite) can have lower tensile strengths (σ) by a factor of ~5 than individual mineral grains (e.g., olivine). Coarse-grained rocks are generally weaker than fine-grained rocks (σ ~ 100 *vs*. 200 MPa, respectively). Thus, fine-grained mantle xenoliths, being stronger, are more likely to be transported intact. Olivine xenocrysts also undergo decompression-induced cracking recorded as sealed and healed cracks (Brett et al. 2015), but require more rapid decompression or greater transport distances to exceed their tensile strengths (~ 500 MPa).

KIMBERLITE TRANSPORT IN DYKES [KIMBERLITE DYKE THEORY]

The main driver for dyke ascent is magma buoyancy, expressed by the buoyancy pressure P_b (Lister and Kerr, 1991):

$$P_b = \Delta\rho\, g\, h \qquad (S2)$$

where Δρ is the density contrast between magma and host-rock, *h* is the ascent distance, and *g* is gravity. When the dyke remains connected to its source at depth and Δρ is 200 kg m^3, the buoyancy pressure along 150 km of dyke will be 300 MPa (Eq. S2).

Dykes are kept open by an elastic overpressure P_e:

$$P_e = \sim \frac{m\, w}{l} \qquad (S3)$$

where *w* is the dyke thickness, *l* is its cross-sectional width, and *m* is the stiffness of the host-material, which is governed by the Young's modulus E and the Poisson's ratio ν:

$$m = \frac{E}{(1 - \nu)} \qquad (S4)$$

For E = 40 GPa, ν = 0.2, and *w/l* of 6 x 10^{-4} (*w* = 0.6 m; *l* = 1 km) the elastic pressure is ~30 MPa.

A tear-drop shaped geometry develops (Takada 1990) once the dyke exceeds the buoyant length L_b (Fig. 4; Taisne and Tait 2009):

$$L_b = \left(\frac{K_c}{\Delta\rho\, g}\right)^{\frac{2}{3}} \qquad (S5)$$

where K_c is the fracture toughness (~ 2 x 10^7 Pa m$^{1/2}$). For proto-kimberlite magma ascending from 150 km depth, the buoyant length of a kimberlite dyke is ~465 m. In the shallow crust this buoyant length would shorten to ~160 m because volatile exsolution will increase Δρ (~1000 kg m^{-3}).

Assuming a tear-drop model geometry (Fig. 4) for kimberlite dykes implies a cross-sectional dyke area of ~400 m^2, assuming *l* is equivalent to L_b, *w* is 1 m and Δρ is 200 kg m^{-3}. The overall volume of the 150 km long dyke would then be ~5x10^7 m^3.

The dynamics of magma flow are expressed by the Reynold's number:

$$Re \equiv \frac{\rho_{liq}\, U\, w}{\mu} \qquad (S6)$$

Where ρ_{liq} is the density of the magma, U is the characteristic velocity, and μ is the magma viscosity. High Reynold's number indicates dominance of inertial forces over viscous forces, and for the case of kimberlite dykes this is due to inferred rapid ascent velocity. The low viscosity of kimberlite melt (< 1 Pa s) means it is highly likely that the flow will have a high Reynold's number and be turbulent. Following Sparks et al. (2006) a characteristic turbulent magma velocity can be calculated for the upward flow of kimberlite:

$$U = 7.7 \left(\frac{(w/2)^5}{\mu\,(\rho_{\mathrm{liq}} g \Delta\rho)^3}\right)^{\frac{1}{7}} g\Delta\rho \qquad \text{(S7)}$$

Where μ is the melt viscosity, ρ_{liq} is the magma density, g is gravity, $\Delta\rho$ is the local density difference, and *w* is the dyke thickness. For proto-kimberlite melt viscosity of 6-36 x 10^{-3} Pa s, magma density of 2800 kg m^{-3}, average dyke width of 0.1-1 m, and density difference of 200-1000 kg m^{-3}, calculated magma velocities are ~20-90 m/s.

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KIMBERLITE MELT PRODUCTION AND TRIGGERING

Here we apply the recent process-oriented models for the segregation and ascent of kimberlite melt. We assume ascent is the consequence of low viscosity buoyant proto-kimberlite melt separating from the asthenosphere and accumulating at the base of the lithosphere. In such circumstances a key boundary is the solidus of the mantle, which for our purposes defines the lithosphere-asthenosphere boundary separating permeable regions with melt present and impermeable regions without melt (Jackson et al. 2018). Melts separating from the asthenosphere by porous flow accumulate as a buoyant melt-rich layer beneath the boundary and we infer that such layers source kimberlites. Time scales for ascent therefore depend on the dimensions of this layer, its growth rate and development of buoyancy-induced instabilities.

We use models of Rayleigh Taylor (RT) instabilities, where a thin buoyant layer of low viscosity melt grows at rate $\dot{h}$ beneath high viscosity mantle lithosphere (Seropian et al. 2018). The fastest growing wavelength, λ, is given by:

$$\lambda = 9.058 \left[\frac{\dot{h}}{g\,\Delta\rho}\right]^{1/2} \mu_1^{-1/6}\,\mu_2^{2/3} \qquad (S8)$$

where μ_1 is the viscosity of the melt layer (~3 x 10^{-2} Pa s; Dobson et al. 1996), μ_2 is the mantle lithosphere viscosity and $\Delta\rho$ is the density difference between the melt layer and overlying mantle. Mantle viscosity depends strongly on temperature, OH-content and mineralogy, and we adopt a value ~$10^{20\pm1}$ Pa s. We assume $\Delta\rho$ of 300 kg/m^3. Approximate values of $\dot{h}$ for the supply of proto-kimberlite melt derive from two plausible conceptual models. If we assume melt results from 0.1% partial melting, a mantle plume with a rise speed of 0.1 m/y (Steinberger and Atretter, 2009) yields a 1-D melt flux ($\dot{h}$) of 10^{-4} m/y and would imply a λ of 11,556 km. Conversely, a passive compaction model for melt extraction (Mackenzie 1985) yields an estimated accumulation rate of ~ 10^{-5} m/year and λ of 3,654 km. However, these wavelengths are much greater than kimberlite cluster footprints and thus it seems that equation (S8) results in geologically and physically unlikely results.

We thus turn to a model (Seropian et al. 2018) of instability from confined layers where the width of the layer is much less than λ. There is a simple linear scaling of the actual horizontal dimension of the buoyant layer, D, to λ with time (see Seropian et al. 2018). A representative time scale for exponential growth is:

$$\tau_c = \frac{\mu_2}{[0.053\, g\, \Delta\rho\, D]} \qquad (S9)$$

and is the time it takes for a small perturbation to grow in height by a factor of e (i.e. ≈ 2.72). The physics of the instability for magma ascent is not, however, captured in equation S9, but we expect τ_c to scale with the instability time through the following arguments. As previously discussed kimberlite ascent through the lithosphere is along dykes, so an RT instability must develop conditions for brittle failure. As an RT instability grows in height there will be an overpressure that increases with height. Further for exponential growth (see Figure 6 in Seropian et al. 2018) tensional strain rate along the melt mantle boundary increases with time. Both of these changes can lead to eventual failure with nucleation and growth of a dyke. A process model of these processes has yet to be developed, but the instability timescale should scale as indicated by equation S9. Thus to get an order of magnitude sense of the frequency of kimberlite ascent and magma volumes we calculate values of τ_c.

A critical layer thickness, h, can be calculated from τ_c and $\dot{h}$ for different values of D. A volume of magma, V_m, is calculated assuming cylindrical geometry from $\pi h(D/2)^2$. Values of D may reflect either ponding in inverted depressions along an irregular topographic base to the lithosphere or be dictated by spacing of deep lithospheric structures that facilitate melt ascent. Based on typical scales of kimberlite vent clusters, and to generate time scales similar to the observed rates, we assumed D values of 300 and 30 km, which return timescales of 68 to 680 ky, respectively. A background melt accumulation rate ($\dot{h}$) of 10^{-5} m/year implies corresponding thicknesses (h) of 0.68 and 6.8 m and volumes of 48 and 4.8 km^3 at these trigger times. These volumes are up to an order of magnitude higher than volumes of individual kimberlites (3.5×10^{-3} to 1 km^3). However, there are many uncertainties in the choices of model input parameters. Furthermore a significant volume (>80%) of the magma generated could be intruded rather than erupted. Thus the difference in results are not regarded as problematic.

Future models of melt layer instability and dyke nucleation need to be developed that can be reconciled with kimberlite erupted volumes as well as rates of kimberlite events.

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